

the closed-loop transfer function is

$$T(s) = \frac{K(K_p s + K_I)}{s^3 + p s^2 + K K_p s + K K_I}. \quad (2.101)$$

We can select the controller gains in Equation (2.101) to place the poles of  $T(s)$  in the desired locations to meet the desired performance specifications. ■

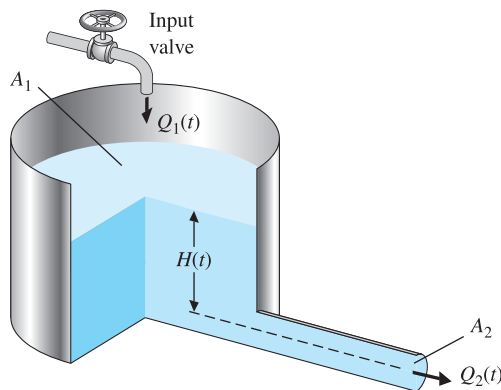
### EXAMPLE 2.12 Fluid flow modeling

A fluid flow system is shown in Figure 2.37. The reservoir (or tank) contains water that evacuates through an output port. Water is fed to the reservoir through a pipe controlled by an input valve. The variables of interest are the fluid velocity  $V$  (m/s), fluid height in the reservoir  $H$  (m), and pressure  $p$  (N/m<sup>2</sup>). The pressure is defined as the force per unit area exerted by the fluid on a surface immersed (and at rest with respect to) the fluid. Fluid pressure acts normal to the surface. For further reading on fluid flow modeling, see [28–30].

The elements of the control system design process emphasized in this example are shown in Figure 2.38. The strategy is to establish the system configuration and then obtain the appropriate mathematical models describing the fluid flow reservoir from an input–output perspective.

The general equations of motion and energy describing fluid flow are quite complicated. The governing equations are coupled nonlinear partial differential equations. We must make some selective assumptions that reduce the complexity of the mathematical model. Although the control engineer is not required to be a fluid dynamicist, and a deep understanding of fluid dynamics is not necessarily acquired during the control system design process, it makes good engineering sense to gain at least a rudimentary understanding of the important simplifying assumptions. For a more complete discussion of fluid motion, see [31–33].

To obtain a realistic, yet tractable, mathematical model for the fluid flow reservoir, we first make several key assumptions. We assume that the water in the tank is incompressible and that the flow is inviscid, irrotational and steady. An incompressible fluid has a constant density  $\rho$  (kg/m<sup>3</sup>). In fact, all fluids are compressible to some extent. The compressibility factor,  $k$ , is a measure of the compressibility of



**FIGURE 2.37**  
The fluid flow reservoir configuration.