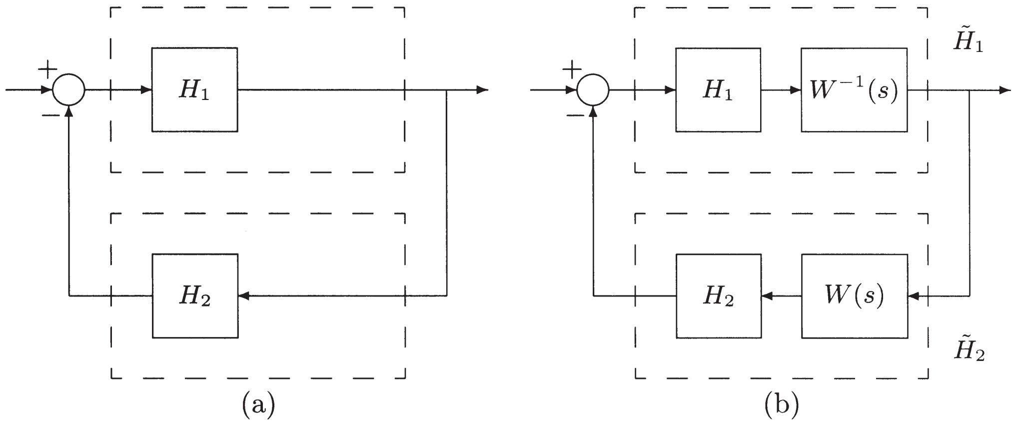

Figure 6.14: Loop transformation with dynamic multipliers.

dynamical system. If postmultiplying H_1 by $(as+1)$ results in a strictly passive system or an output strictly passive system that is zero-state observable, we can employ Theorem 6.3 to conclude asymptotic stability of the origin. This idea is illustrated in the next two examples for cases where H_1 is linear and nonlinear, respectively.

Example 6.15 Let H_1 be a linear time-invariant system represented by the state model

$$\dot{x} = Ax + Be_1, \quad y_1 = Cx$$

where

$$A = \begin{bmatrix} 0 & 1 \\ -1 & -1 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \quad \text{and} \quad C = \begin{bmatrix} 1 & 0 \end{bmatrix}$$

Its transfer function $1/(s^2+s+1)$ has relative degree two; hence, it is not positive real. Postmultiplying H_1 by $(as+1)$ results in $\tilde{H}_1$, which can be represented by the state model

$$\dot{x} = Ax + Be_1, \quad \tilde{y}_1 = \tilde{C}x$$

where $\tilde{C} = C + aCA = \begin{bmatrix} 1 & a \end{bmatrix}$. Its transfer function $(as+1)/(s^2+s+1)$ satisfies the conditions

$$Re\left[\frac{1+j\omega a}{1-\omega^2+j\omega}\right] = \frac{1+(a-1)\omega^2}{(1-\omega^2)^2+\omega^2} > 0, \quad \forall\, \omega \in R$$

and

$$\lim_{\omega\to\infty} \omega^2\, Re\left[\frac{1+j\omega a}{1-\omega^2+j\omega}\right] = a - 1 > 0$$

if $a > 1$. Thus, choosing $a > 1$, we can apply Lemmas 6.1 and 6.4 to conclude that $\tilde{H}_1$ is strictly passive with the storage function $(1/2)x^TPx$ where P satisfies the equations

$$PA + A^TP = -L^TL - \varepsilon P, \quad PB = \tilde{C}^T$$