Lesson 1 Periodic Signals

1.1 Time-Domain Description

The fact that great majority of functions which may usefully be considered as signals are functions of time lends justification to the treatment of signal theory in terms of time and of frequency. A periodic signal will therefore be considered to be one which repeats itself exactly every T seconds, where T is called the period of the signal waveform; the theoretical treatment of periodic waveforms assumes that this exact repetition is extended throughout all time, both past and future. In practice, of course, signals do not repeat themselves indefinitely. Nevertheless, a waveform such as the output voltage of a main rectifier prior to smoothing does repeat itself very many times, and it analysis as a strictly periodic signal yields valuable results.[1] In other cases, such as the electrocardiogram, the waveform is quasi-periodic and may usefully be treated as truly periodic for some purpose. It is worth nothing that a truly repetitive signal is of very little interest in a communication channel, since no further information is conveyed after the first cycle of the waveform has been received. One of the main reasons for discussing periodic signals is that a clear understanding of their analysis is a great help when dealing with periodic and random ones.

A complete time-domain description of such a signal involves specifying its value precise at every instant of time. In some cases this may be done very simply using mathematical notation. Fortunately, it is in many cases useful to describe only certain aspects of a signal waveform, or to represent it by a mathematical formula which is only approximate. The following aspects might be relevant in particular cases.

(1) the average value of the signal.

(2) the peak value reached by the signal.

(3) the proportion of the total time spent between value a and b.

(4) the period of the signal.

If it is desired to approximate the waveform by a mathematical expression, such as a polynomial expansion, a Taylor series, or a Fourier series may be used. A polynomial of order n having the form

$$f(t) = a_0 + a_1 t + a_2 t^2 + a_3 t^3 + \cdots + a_n t^n \tag{1-1}$$

may be used to fit the actual curve at $(n+1)$ arbitrary points. The accuracy of fit will generally improve as number of polynomial terms increases. It should also be noted that the error figure between the true signal waveform and the polynomial will normally become very large away from the region of the fitted points, and that the polynomial itself cannot be periodic. Whereas a polynomial approximation fits the actual waveform at a number of arbitrary points, the alternative Taylor series approximation provides a good fit to a smooth continuous waveform in the vicinity of one selected point. The coefficients of the Taylor series are chosen to make the series and its derivatives agree with the actual waveform at this point. The number of terms in the series determines to what order of derivative this agreement will extend, and hence the accuracy with which series and actual waveform agree in the region of point chosen. The general form of the Taylor series for approximating a function in the region of the point is given by

$$f(t) = f(a) + (t-a) \cdot \frac{\mathrm{d}f(a)}{\mathrm{d}t} + \frac{(t-a)^2}{2!} \cdot \frac{\mathrm{d}^2 f(a)}{\mathrm{d}t^2} + \cdots + \frac{(t-a)^n}{n!} \cdot \frac{\mathrm{d}^n f(a)}{\mathrm{d}t^n} \quad (1\text{-}2)$$

Generally speaking, the fit to the actual waveform is good in the region of the point chosen, but rapidly deteriorates to either side. The polynomial and Taylor series descriptions of a signal waveform are therefore only to be recommended when one is concerned to achieve accuracy over a limited region of the waveform. The accuracy usually decreases rapidly outside this region, although it may be improved by including additional terms (so long as t lies within the region of convergence of the series). [2] The approximations provided by such methods are never periodic in form and cannot therefore be considered ideal for the description of repetitive signals.

By contrast the Fourier series approximation is well suited to the representation of a signal waveform over an extended interval. When the signal is periodic, the accuracy of the Fourier series description is maintained for all time, since the signal is represented as the sum of a number of sinusoidal functions, which are themselves periodic. Before examining in detail the Fourier series method of representing a signal, the background to what is known as the 'frequency-domain' approach will be introduced.

1.2 Frequency-Domain Description

The basic conception of frequency-domain analysis is that a waveform of any complexity may be considered as the sum of a number of sinusoidal waveforms of suitable amplitude, periodicity, and relative phase. [3] A continuous sinusoidal function ($\sin \omega t$) is thought of as a 'single frequency' wave of frequency radians per second, and the frequency-domain description of a signal involves its breakdown into a number of such basic functions. This is the method of Fourier analysis.

There are a number of reasons why signal representation in terms of a set of component sinusoidal waves occupies such a central role in signal analysis. The suitability of a set of periodic functions for approximating a signal waveform over an extended interval has already been mentioned, and it will be shown later that the use of such techniques causes the error between the actual signal and its approximation to be minimized in a certain important sense. A further reason why sinusoidal functions are so important in signal analysis is that they occur widely in the physical world and are very susceptible to mathematical treatment; a large and extremely important class of electrical and mechanical systems, known as 'linear systems', responds sinusoidally when driven by a sinusoidal disturbing function of any frequency. All these manifestations of sinusoidal function in the physical world suggest that signal analysis in sinusoidal terms will simplify the problem of relating a signal to underlying physical causes, or to the physical properties of a system or device through which it has passed. Finally, sinusoidal functions form a set of what are called 'orthogonal function', the rather special properties and advantage of which will now be discussed.

1.3 Orthogonal Functions

1.3.1 Vectors and Signals

A discussion of orthogonal functions and of their value for the description of signals may be conveniently introduced by considering the analogy between vectors and signals. A vector is specified both by its magnitude and direction, familiar examples being force and velocity. Suppose we have two $\boldsymbol{V}_1$ and $\boldsymbol{V}_2$; geometrically, we define the component of vector $\boldsymbol{V}_1$ along vector $\boldsymbol{V}_2$ by constructing the perpendicular form the end of $\boldsymbol{V}_1$ onto $\boldsymbol{V}_2$. We then have

$$\boldsymbol{V}_1 = C_{12}\boldsymbol{V}_2 + \boldsymbol{V}_e \tag{1-3}$$

where vector $\boldsymbol{V}_e$ is the error in the approximation. Clearly, this error vector is of minimum length when it is drawn perpendicular to the direction of $\boldsymbol{V}_2$. Thus we say that the component of vector $\boldsymbol{V}_1$ along vector $\boldsymbol{V}_2$ is given by $C_{12}\boldsymbol{V}_2$, where C_{12} is chosen such as to make the error vector as small as possible. A familiar case of an orthogonal vector system is the use of three mutually perpendicular axes in co-ordinate geometry.

There basic ideas about the comparison of vectors may be extended to signals. Suppose we wish to approximate a signal $f_1(t)$ by another signal or function $f_2(t)$ over a certain interval $t_1<t<t_2$; in other words,

$$f_1(t) \approx C_{12} f_2(t) \qquad \text{for} \quad t_1 < t < t_2$$

We wish to choose C_{12} to achieve the best approximation. If we define the error function

$$f_e(t) = f_1(t) - C_{12} f_2(t) \tag{1-4}$$

it might appear at first sight that we should choose C_{12} so as to minimize the average value of $f_e(t)$ over the chosen interval. The disadvantage of such an error criterion is that large positive and negative errors occurring at different instants would tend to cancel each other out. This difficulty is avoided if we choose to minimize the average squared-error, rather than the error itself (this is equivalent to minimizing the square root of the mean-squared error, or 'r. m. s' error). Denoting the average of $f_e^2(t)$ by ε, we have

$$\varepsilon = \frac{1}{(t_2 - t_1)}\int_{t_1}^{t_2} f_e^2(t)\mathrm{d}t = \frac{1}{(t_2 - t_1)}\int_{t_1}^{t_2} [f_1(t) - C_{12} f_2(t)]^2 \mathrm{d}t \tag{1-5}$$

Differentiating with respect to C_{12} and putting the resulting expression equal to zero gives the value of C_{12} for which is a minimum.[4] Thus

$$\frac{\mathrm{d}}{\mathrm{d}C_{12}}\left\{\frac{1}{(t_2 - t_1)}\int_{t_1}^{t_2} [f_1(t) - C_{12} f_2(t)]^2 \mathrm{d}t\right\} = 0$$

Expanding the bracket and changing the order of integration and differentiating gives

$$C_{12} = \int_{t_1}^{t_2} f_1(t) f_2(t)\mathrm{d}t \bigg/ \int_{t_1}^{t_2} f_2^2(t)\mathrm{d}t \tag{1-6}$$

1.3.2 Signal Description by Sets of Orthogonal Function

Suppose that we have approximated a signal $f_1(t)$ over a certain interval by the function

$f_2(t)$ so that the mean square error is minimized, but that we now wish to improve the approximation. It will be demonstrated that a very attractive approach is to express the signal in terms of a set of function $f_2(t)$, $f_3(t)$, $f_4(t)$, etc., which are mutually orthogonal. Suppose the initial approximation is

$$f_1(t) \approx C_{12} f_2(t) \tag{1-7}$$

and that the error is further reduced by putting

$$f_1(t) \approx C_{12} f_2(t) + C_{13} f_3(t) \tag{1-8}$$

where $f_2(t)$ and $f_3(t)$ are orthogonal over the interval of interest. Now that we have incorporated the additional term $C_{13} f_3(t)$, it is interesting to find what the new value of must be in order that the mean square error is again minimized. We now have

$$f_e(t) = f_1(t) - C_{12} f_2(t) - C_{13} f_3(t) \tag{1-9}$$

and the mean square error in the interval $t_1 < t < t_2$ is therefore

$$\varepsilon = \frac{1}{t_2 - t_1} \cdot \int_{t_1}^{t_2} [f_1(t) - C_{12}(t) f_2(t) - C_{13}(t) f_3(t)]^2 \mathrm{d}t \tag{1-10}$$

Differentiating partially with respect to C_{12} to find the value of C_{12} for which the mean square error is again minimized, and changing the order of differentiation and integration, we have again [5]

$$C_{12} = \int_{t_1}^{t_2} f_1(t) f_2(t) \mathrm{d}t \Big/ \int_{t_1}^{t_2} f_2^2(t) \mathrm{d}t \tag{1-11}$$

In order words, the decision to improve the approximation by incorporating an additional term in does not require us to modify the coefficient, provided that $f_3(t)$ is orthogonal to $f_2(t)$ in the chosen time interval.[6] By precisely similar arguments we could show that the value of C_{13} would be unchanged if the signal was to be approximated by $f_3(t)$ alone.

This important result may be extended to cover the representation of a signal in terms of a whole set of orthogonal functions. The value of any coefficient does not depend upon how many functions from the complete set are used in the approximation, and is thus unaltered when further terms are included.[7] The use of a set of orthogonal functions for signal description is analogous to the use of three mutually perpendicular (that is, orthogonal) axes for the description of a vector in three-dimensional space, and gives rise to the notion of a 'signal space'.[8] Accurate signal representation will often require the use of many more than three orthogonal functions, so that we must think of a signal within some interval $t_1 < t < t_2$ as being represented by a point in a multidimensional space.

To summarize, there are a number of sets of orthogonal functions available such as the so-called Legendre polynomials and Walsh functions for the approximate description of signal waveform, of which the sinusoidal set is the most widely used.[9] Sets involving polynomials in t are not by their very nature periodic, but may sensibly be used to describe one cycle (or less) of a periodic waveform; outside the chosen interval, errors between the true signal and its approximation will normally increase rapidly. A description of one cycle of a periodic signal in terms of sinusoidal function will, however, be equally valid for all time because of the every member of the orthogonal.

1.4 The Fourier Series

The basis of the Fourier series is that complex periodic waveform may be analyzed into a number of harmonically related sinusoidal waves which constitute an orthogonal set. If we have a periodic signal $f(t)$ with a period equal to T, then $f(t)$ may be represented by the series

$$f(t)=A_0+\sum_{n=1}^{\infty}A_n\cos n\omega_1 t+\sum_{n=1}^{\infty}B_n\sin n\omega_1 t \tag{1-12}$$

where $\omega_1=2\pi/T$. Thus $f(t)$ is considered to be made up by the addition of a steady level A_0 to a number of sinusoidal and cosinusoidal waves of different frequencies. The lowest of these frequencies is ω_1 (radians per second) and is called the 'fundamental'; waves of this frequency have a period equal to that of the signal. Frequency $2\omega_1$ is called the 'second harmonic', $3\omega_1$ is the 'third harmonic', and so on. Certain restrictions, known as the Dirichlet conditions, must be placed upon $f(t)$ for the above series to be valid. The integral $\int |f(t)|\,\mathrm{d}t$ over a complete period must be finite, and may not have more than a finite number of discontinuities in any finite interval. Fortunately, these conditions do not exclude any signal waveform of practical interest.

1.4.1 Evaluation of the Coefficients

We now turn to the question of evaluating the coefficients A_0, A_n and B_n. Using the minimum square error criterion described in foregoing text, and writing for the sake of convenience, we have

$$\left.\begin{aligned} A_0 &= \frac{1}{2\pi}\int_{-\pi}^{\pi} f(x)\,\mathrm{d}x \\ A_n &= \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos nx\,\mathrm{d}x \\ B_n &= \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin nx\,\mathrm{d}x \end{aligned}\right\} \tag{1-13}$$

Although in the majority of cases it is convenient for the interval of integration to be symmetrical about the origin, any interval equal in length to one period of the signal waveform may be chosen.

Many waveforms of practical interest are either even or odd functions of time. If $f(t)$ is even then by definition $f(t)=f(-t)$, whereas if it is odd $f(-t)=-f(t)$. If $f(t)$ is even and we multiply it by the odd function $\sin n\omega_1 t$ the result is also odd. Thus the integrand for every B_n is odd. Now when an odd function is integrated over an interval symmetrical about $t=0$, the result is always zero. Hence all the B coefficients are zero and we are left with a series containing only cosines. By similar arguments, if $f(t)$ is odd the A coefficients must be zero and we are left with a sine series. It is indeed intuitively clear that an even function can only be built up from a number of other functions which are themselves even, and vice versa.

We have already seen how the Fourier series is simplified in the case of an even or odd function, by losing either its sine or its cosine terms. A different type of simplification occurs in the case of a waveform possessing what is know as 'half-wave symmetry'. In mathematical terms, half-wave symmetry exists when

$$f(t) = -f(t+T/2) \tag{1-14}$$

In other words, any two values of the waveform separated by $T/2$ will be equal in magnitude and opposite in sign. Generalizing, only odd harmonics exhibit half-wave symmetry, and therefore a waveform of any complexity which has such symmetry cannot contain even harmonic components. Conversely, a waveform know to contain any second, fourth, or other harmonic components cannot display half-wave symmetry.

Usually, we have always integrated over a complete cycle to derive the coefficients. However in the case of an odd or even function it is sufficient, and often simpler, to integrate over only one half of the cycle and multiply the result by 2. Furthermore if the wave is not only even or odd but also display half-wave symmetry, it is enough to integrate over one quarter of a cycle and multiply by 4. These closer limits are adequate in such cases the function that is being integrated is repetitive, repeating twice within one period when it also exhibit half-wave symmetry.

1.4.2 Choice of Time Origin, and Waveform Power

The amount of work involved in calculating the Fourier series coefficients for a particular waveform shape is reduced if the waveform is either even or odd, and this may often be arranged by a judicious choice of time origin (that is, shift of time origin).[10] This shift has therefore merely had the effect of converting a Fourier series containing only sine terms into one containing only cosine terms; the amplitude of a component at any one frequency is, as we would expect, unaltered. For a complicated waveform which is neither even nor odd, it must be expected to include both sine and cosine terms in its Fourier series.

As the time origin of a waveform is shifted, the various sine and cosine coefficients of its Fourier series will change, but the sum of the squares of any two coefficients A_n and B_n will remain constant, which means that the average power of the waveform, a concept familiar to electrical engineers, is unaltered.

The above ideas lead naturally to an alternative trigonometric of the Fourier series. If the two fundamental components of a waveform are

$$A_1 \cos \omega_1 t \quad \text{and} \quad B_1 \sin \omega_1 t$$

their sum may be expressed in an alternative form using trigonometric identities

$$\begin{aligned} A_1 \cos \omega_1 t + B_1 \sin \omega_1 t &= \sqrt{(A_1^2 + B_1^2)} \cos\left(\omega_1 t - \arctan \frac{B_1}{A_1}\right) \\ &= \sqrt{(A_1^2 + B_1^2)} \sin\left(\omega_1 t + \arctan \frac{B_1}{A_1}\right) \end{aligned} \tag{1-15}$$

Thus the sine and cosine components at a particular frequency are expressed as a single cosine or sine wave together with a phase shift. If this procedure is applied to all harmonic components of the Fourier series, we get the alternative forms

$$f(t) = A_0 + \sum_{n=1}^{\infty} C_n \cos(n\omega_1 t - \phi_n) \quad \text{or} \quad f(t) = A_0 + \sum_{n=1}^{\infty} C_n \sin(n\omega_1 t + \theta_n) \qquad (1\text{-}16)$$

where

$$C_n = \sqrt{A_n^2 + B_n^2}, \phi_n = \arctan(B_n/A_n), \theta_n = \arctan(A_n/B_n) \qquad (1\text{-}17)$$

Finally, we note that sine the mean power represented by any component wave is

$$(A_n^2 + B_n^2)/2 = C_n^2/2 \qquad (1\text{-}18)$$

and the power represented by the term A_0 is simply A_0^2, the total average waveform power is equal to

$$P = A_0^2 + \frac{1}{2}\sum_{n=1}^{\infty} C_n^2 \qquad (1\text{-}19)$$

But P may be expressed as the average value over one period of $[f(t)]^2$, using again the convention that is considered to represent a voltage waveform applied across a ohm resistor. Hence

$$P = A_0^2 + \frac{1}{2}\sum_{n=1}^{\infty} C_n^2 = \frac{1}{T}\int_{-T/2}^{T/2} [f(t)]^2 \mathrm{d}t \qquad (1\text{-}20)$$

This result is a version of a more general one known as Parseval's theorem, and shows that the total waveform power is equal to the sum of the powers represented by its individual Fourier components. It is, however, important to note that this is only true because the various component waves are drawn from an orthogonal set.

Words and Expressions

accuracy [ˈækjurəsi] *n.* 精确性，准确度，精度
amplitude [ˈæmplitjuːd] *n.* 振幅，幅度
aperiodic [ˈeipiəriˈɔdik] *adj.* 非周期的
approach [əˈprəutʃ] *n.* 接（逼）近；近似法（值）；途径，方法
approximation [əˌprɔksiˈmeiʃən] *n.* 近似值
arbitrary [ˈɑːbitrəri] *adj.* 任意的
channel [ˈtʃænl] *n.* 信道，频道
coefficient [ˌkəuiˈfiʃənt] *n.* 系数
convergence [kənˈvəːdʒəns] *n.* 收敛
conversely [kənˈvəːsli] *adj.* 相反的，逆的
coordinate [kəuˈɔːdinit] *n.* 坐标（系）
criterion [kraiˈtiəriən] *n.* 标准，规范
deteriorate [diˈtiəriəreit] *vi.* 恶化，变坏，退化
differentiate [ˌdifəˈrenʃieit] *vt.* 求……的微分
dimension [diˈmenʃən] *n.* 维数
Dirichlet conditions 狄利克雷条件
discontinuity [ˈdisˌkəntiˈnju（ː）iti] *n.* 心电图，心动电流图
even [ˈiːvən] *adj.* 偶数的

expansion [iks'pænʃən] *n.* 展开（式）

foregoing [fɔː'ɡəuiŋ] *adj.* 前述的，在前的

geometrical [dʒiə'metrikəl] *adj.* 几何学的

half-wave symmetry　半波对称

harmonical [hɑː'mɔnikəl] *adj.* 谐波的

identity [ai'dentiti] *n.* 恒等式

instant ['instənt] *n.* 瞬时，瞬间

integrand ['intiɡrænd] *n.* 被积函数

integrate ['intiɡreit] *vt.* 求……的积分

intuitively [in'tju（:）itivli] *adv.* 直观地，直觉地

geometry [dʒi'ɔmitri] *n.* 几何学

Legendre polynomials　勒让德多项式

linear ['liniə] *adj.* 线性的

main [mein] *n.* 电源，电力网

manifestation [ˌmænifes'teiʃən] *n.* 表现

minimum ['miniməm] *n.* 最小值，最小化

mutually ['mjuːtʃuəli] *adv.* 相互地

notation [nəu'teiʃən] *n.* 符号，记号

odd [ɔd] *adj.* 奇数的，单数的

ohm [əum] *n.* 欧姆

order ['ɔːdə] *n.* 次序，阶

origin ['ɔridʒin] *n.* 原点

orthogonal [ɔː'θɔɡənl] *adj.* 正交的，直角的

peak [piːk] *adj.* 最高的；*n.* 最高峰

periodicity [ˌpiəriə'disiti] *n.* 周期

perpendicular [ˌpəːpən'dikjulə] *adj.* 垂直的，正交的

phase [feiz] *n.* 相位

polynomial [ˌpɔli'nəumjəl] *adj.* 多项式的；*n.* 多项式

quasi-periodic　准周期的

radian ['reidjən] *n.* 弧度

rectifier ['rektifaiə] *n.* 整流器

resistor [ri'zistə] *n.* 电阻器

series ['siəriːz] *vt.* 展成级数；*n.* 级数

set [set] *n.* 集合

sinusoidal [ˌsainə'sɔidəl] *adj.* 正弦的

susceptible [sə'septəbl] *adj.* 敏感的，易受影响的

symmetry ['simitri] *n.* 对称

symmetrical [si'metrikəl] *adj.* 对称的，均匀的

term [təːm] *n.* 术语，（数）项

theorem [ˈθiərəm] *n.* 定理，法则
time domain　时域
trigonometric [ˌtrigənəˈmetrik] *adj.* 三角法的
trigonometric identities　三角恒等式
vector [ˈvektə] *n.* 矢量，向量
vice versa　反之亦然
vicinity [viˈsiniti] *n.* 附近
Walsh function　沃尔什函数

Notes

1. Nevertheless，a waveform such as the output voltage of a main rectifier prior to smoothing does repeat itself very many times，and it analysis as a strictly periodic signal yields valuable results.
   不过，像电源整流器输出电压这样的波形，在平滑之前，还是重复本身很多次的，将其作为严格的周期信号进行分析，会产生颇有价值的结果。
2. The accuracy usually decreases rapidly away from this region，although it may be improved by including additional terms（so long as t lies within the region of convergence of the series).
   在所选择区域之外，精度通常会迅速降低，尽管可以通过补充一些项，使之有所改善（只要 t 位于序列的收敛域内）。
3. The basic conception of frequency-domain analysis is that a waveform of any complexity may be considered as the sum of a number of sinusoidal waveforms of suitable amplitude，periodicity，and relative phase.
   频域分析的基本概念是：任何复杂波形都可以看成许多具有适当振幅、周期和相对相位的正弦波之和。
4. Differentiating with respect to C_{12} and putting the resulting expression equal to zero gives the value of C_{12} for which ε is a minimum.
   对 C_{12} 求微分，然后令所得表达式为 0，就可以得到使 ε 最小的 C_{12} 值。
5. Differentiating partially with respect to C_{12} to find the value of C_{12} for which the mean square error is again minimized，and changing the order of differentiation and integration，we have again aquation（1-11).
   为了求出使均方误差仍保持最小的 C_{12} 值，先对 C_{12} 求偏微分，再交换微分与积分次序，我们再次得到式（1-11）。
6. In order words，the decision to improve the approximation by incorporating an additional term in does not require us to modify the coefficient C_{12}，provided that $f_3(t)$ is orthogonal to $f_2(t)$ in the chosen time interval.
   换言之，如果 $f_2(t)$与 $f_3(t)$在所选择的时间区间内正交，在并入用 $f_3(t)$表示的附加项来改善逼近程度时，系数 C_{12} 无须修正。

7. This important result may be extended to cover the representation of a signal in terms of a whole set of orthogonal functions. The value of any coefficient does not depend upon how many functions from the complete set are used in the approximation, and is thus unaltered when further terms are included.

   这个重要结论可以推广到用整个正交函数集表示信号的情况。在进行逼近时，任何系数值与完备集合中用了多少函数没有关系，因此函数包含更多的项时，这些系数值不会改变。

8. The use of a set of orthogonal functions for signal description is analogous to the use of three mutually perpendicular (that is, orthogonal) axes for the description of a vector in three-dimensional space, and gives rise to the notion of a 'signal space'.

   利用一个正交函数集描述信号，类似于在三维空间中利用三个互相垂直的轴描述矢量，这就引出了“信号空间”的概念。

9. To summarize, there are a number of sets of orthogonal functions available such as the so-called Legendre polynomials and Walsh functions for the approximate description of signal waveform, of which the sinusoidal set is the most widely used.

   总之，有许多正交函数集可用来近似描述信号波形，如所谓的勒让德多项式和沃尔什函数等，正弦函数集是其中最常用的。

10. The amount of work involved in calculating the Fourier series coefficients for a particular waveform shape is reduced if the waveform is either even or odd, and this may often be arranged by a judicious choice of time origin (that is, shift of time origin).

    对于一个具体的波形而言，如果它是偶函数或奇函数，那么计算该波形傅里叶级数系数时，可以通过适当的选择时间原点来减小其计算工作量。

Exercises

Translate the following passages into English or Chinese.

1. 简谐信号是最简单和最重要的周期信号。任意一个周期信号都可以用简谐信号来表达，两者之间联系的桥梁是傅里叶级数，所以傅里叶级数是周期信号分析的理论基础。
2. 一个在时域上显得很复杂的信号，将其变换或映射到频域（包括 s 和 z 域），就能够分解为非常简单的基本信号形式以进行分析和求解。
3. 频谱是由不连续的谱线组成的，每条谱线代表一个谐波分量，这种频谱称为离散频谱。谱线之间的间隔等于基波频率 ω_0 的整数倍。即频谱中的每一条谱线只能出现在基波频率 ω_0 的整数倍上，各谐波的频率 $n\omega_0$ 都是基波频率的整数倍。
4. The Fourier series is a particular type of orthogonal series representation that is very useful in solving engineering problems, especial communication problems. The orthogonal functions that are used are either sinusoids, or, equivalently, complex exponential functions.
5. For periodic waveforms, the Fourier series representations are valid over all time. Consequently, the (two-side) spectrum, which depends on the waveshape from $t=-\infty$ to $t=\infty$, may be evaluated in terms of Fourier coefficients.

Reading Material

Underwater Acoustic Signals

In the operation of a sonar system the operator is repeatedly faced with the problem of detecting a signal which is obscured by noise. This signal may be an echo resulting from a transmitted signal over which the operator has some control, or it may have its origin in some external source. These two modes of operation are commonly distinguished as active and passive sonar, respectively. Similar situations arise in radar surveillance and in disciplines for techniques and for illustrations of the basic principles.

Since there are many ways in which one can think about signal detection, it is desirable to define a term to denote special cases. The word detection will be used when the question to be answered is, 'Are one or more signals present?' when the system is designed to provide an answer to this question, either deterministic or probabilistic, one speaks of hypothesis testing. The case of a single signal occurs so often that many systems are designed to provide only two answers, 'Yes, a signal is present,' or 'No, there is no signal.' One can make the problem more complicated by endeavoring to classify the signal into categories. Decisions of this latter kind will be referred to as target classification.

Normally a piece of detection equipment is designed to operate in a fixed mode and the parameters such as integrating time of rectifier circuits or persistence of the oscilloscope tube for visual detection cannot be changed readily. There will always be some uncertain signals, which the observer will be hesitant to reject or accept. In these cases the operator might have the feeling that if the integrating time of the detector or the persistence of the oscilloscope tube were longer, he could reach a decision about the existence of the signal. Wald (1950) has formulated this intuitive feeling into a theory of detection. When one is able to vary deliberately the interval over which one stores data in the reception system in order to achieve a certain level of certainty, one speaks of sequential detection.

Frequently it is desirable to determine not only the presence or absence of the signal but also one or more parameters associated with the signal. The parameters of interest can vary widely from a simple quantity such as time of arrival or target bearing to the recovery of the complete waveform. When a system is designed to recover one or more parameters associated with the signal, one speaks of signal extraction.

The word signal was not defined and it was assumed that the reader had an intuitive felling for the word. Some elaboration may be in order since the definition of signal as subjective and depends on the application. One may say that 'signal' is what one wants to observe and noise is anything that obscures the observation. Thus, a tuna fisherman who is searching the ocean with the aid of sonar equipment will be overjoyed with sounds that are impairing the performance of a nearby sonar system engaged in tracking a submarine. Quite literally, one man's signal is another man's noise.

Signals come in all shapes and forms. In active sonar system one may use simple sinusoidal signals of fixed duration and modulations thereof. There are impulsive signals such as those made with explosions or thumpers. At the other extreme one may make use of pseudorandom signals. In passive systems, the signals whose detection is sought may be noise in the conventional meaning of the word; noise produced by propellers or underwater swimmers, for example. It should be evident that one of our problem will be the formulation of mathematical techniques that can be used to describe the signal.

Although the source in an active sonar search system may be designed to transmit a signal known shape, there is no guarantee that the return signal whose detection is sought will be similar. In fact, there are many factors to change the signal. The amplitude loss associated with inverse spherical spreading is most unfortunate for the detection system nut it does not entail any distortion of the wave shape. (Incidentally, this happy state of affairs does not apply to two-dimensional waves except in the far field where the wave can be approximated locally as a plane wave.) The acoustic medium has an attenuation factor, which depends on the frequency. This produces a slight distortion of the wave shape and a corresponding change in the energy spectrum of the pulse. The major changes in the waveform result from acoustic boundaries and inhomogeneities in the medium.

When echoes are produced by extended targets such as submarines, there are two distinct ways in which the echo structure is affected. First, there is the interference between reflections from the different leads to a target strength that fluctuates rapidly with changes in the aspect. Secondly, there is the elongation of the composite echo due to the distribution of reflecting features along the submarines. This means that the duration of the composite echo is dependent in a simple manner on the aspect angle. If T is the duration of the echo from a point scatterer, and L is the length of the submarine, the duration of the returned echo will be $T+(2L/C)\cos\theta$, where θ is the acute angle between the major axis of the submarine and the line joining the source and the submarine. C is the velocity of sound in the water. Of course, $L\cos\theta$ must be replaced by the beam width of the submarine when θ is near.

A final source of pulse distortion is the Doppler shifts produced by the relative motions between the source, the medium, the bottom, and the targets. Since the source, the medium, and the target (or detector in passive listening) may each have a different vector velocity relative to the bottom, the variety of effects may be quite large.